RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2019

SECOND YEAR [BATCH 2018-21]

Date : 21/12/2019 **MATHEMATICS FOR ECONOMICS [General]**

Time : 11 am – 2 pm **Paper : III** Full Marks : 75

[Use a separate Answer Book for each Group]

Group – A

Answer any four questions from Question nos. 1 to 6 : [4×5]

1. Let , $f(x,y)=\begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2} & ,\, when\, x^2+y^2\neq 0 \\ 0 & ,\, when\, x^2+y^2=0 \end{cases}$

   Show that f_y exist at (0,0) and f_x is continuous at (0,0) . What so you say about the differentiability of the function f at (0,0) ?

2. State and prove Euler's theorem on homogeneous function for two variables. [5]

3. Find the directional derivative of $f(x,y,z)=x^2+y^2+xz$ at the point (2,1,0) in the direction of the vector (1,1,1). [5]

4. (a) Find $\dfrac{\partial z}{\partial r}$ *and* $\dfrac{\partial z}{\partial \theta}$, if $z=x^2+xy+y^2$ and $x=2r+\theta$, $y=r-2\theta$.

   (b) Check whether (0,0) is a saddle point of the function f(x , y) = $x^2-3xy^2+2y^4$.

5. (a) Evaluate : $\int_0^2 \dfrac{dx}{\sqrt{x(2-x)}}$.

   (b) State the Fundamental theorem of Integral Calculus .

6. (a) Show that $(-1,0)$ is a point of inflexion of $y=\dfrac{x|x+1|}{x+2}$, $x\neq -2$. [3]

   (b) Determine whether $f(x)=\log_e x$ is convex or concave in $(0,\infty)$. [2]

Answer any one question from Question nos. 7 to 9 : [2×10]

7. (a) Evaluate : $\lim\limits_{n\to\infty}\left[\dfrac{n+1}{n^2+1^2}+\dfrac{n+2}{n^2+2^2}+\cdots\cdots+\dfrac{1}{n}\right]$

   (b) If the variables x , y , z satisfy the equation $\phi(x)\phi(y)\phi(z)=k^3$ *and* $\phi(a)=k,$

   $\phi'(a)\neq 0$. Then prove that , $f'(a)\left\{\dfrac{\phi''(a)}{\phi'(a)}-\dfrac{\phi'(a)}{\phi(a)}\right\}>f''(a)$ if the function $f(x)+f(y)+f(z)$ has a maximum at $x=y=z=a$.

8. (a) (i) State the relation between Beta function and Gamma function . Using this relation find $\Gamma\left(\frac{1}{2}\right)$.

(ii) Show that $\int_0^\infty x^2 e^{-x^4}\,dx \times \int_0^\infty e^{-x^4}\,dx = \frac{\pi}{8\sqrt{2}}$. (Using the result $\Gamma(m)\Gamma(1-m) = \frac{\pi}{\sin m\pi}$,$0 < m < 1$).

(b) (i) State implicit functions theorem for a real value continuous function of two variables .

(ii) $f(x, y) = x^2 + xy + y^2 - 1 = 0$ in the neighbourhood of (0 , -1) . Find implicit function in term of x i.e $y = \phi(x)$.

9. Determine the maxima and minima of $f(x, y, z) = x + y + 2z$ on the surface $x^2 + y^2 + z^2 = 3$, using the Lagrange's method of undetermined multipliers . [10]

Group – B

***Answer any seven questions from Question Nos. 10 to 20* :** [7×5]

10. Show that the differential equation $\frac{dy}{dx} + P(x)y = Q(x)y^n$ can be reduced to a linear differential equation using the substitution $z = y^{n-1}$. [5]

11. Show that $e^{\int P(x)dx}$ is an integrating factor of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$. [5]

12.(a) Write down the order and degree of the following differential equation

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]\frac{d^3y}{dx^3} - 3\frac{dy}{dx}\frac{d^2y}{dx^2} = 0 .$$ [1+1]

(b) Find the differential equation of the family of circles touching the x-axis at the origin. [3]

13. Reduce the following equation to homogeneous form and hence solve it :

$$\frac{dy}{dx} = \frac{2x - y + 1}{6x - 5y + 4} .$$ [5]

14.(a) Define the Clairaut's form of a first order higher degree differential equation. [2]

(b) Reduce the differential equation to the Clairaut's form and hence find the complete Primitive. $\sin px \cos y = \cos px \sin y + y$, where $p = \frac{dy}{dx}$. [3]

15. Apply the method of variation of parameters to solve the differential equation

$$\frac{d^2y}{dx^2} + a^2 y = \tan ax .$$ [5]

16. Solve the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 2x^2 + 1$ by the method of undetermined coefficient. [5]

17. Transform the following single linear differential equation into a system of first order differential equation $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = t^2$. [5]

18. Solve : $(x^2 y - 2xy^2)dx + (3x^2 y - x^3)dy = 0$. [3+2]

19. Solve : $y = px + \sqrt{a^2 p^2 + b^2}$,where $p = \frac{dy}{dx}$. [5]

20. Find the general solution of the following system of linear differential equations. [5]

$$\frac{dx_1}{dt} = 3x_1 + 4x_2 - 2x_3$$

$$\frac{dx_2}{dt} = 2x_1 + x_2 - 4x_3$$

$$\frac{dx_3}{dt} = x_1 + 2x_2$$

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